Not just True or False

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What is a Valuation?

- **Proposition** = Statement regarding some properties of a system: x=3, $A \in \Delta$
- **Valuation** = Assignment of truth values to propositions, i.e. assessing whether the proposition is true or false.
- **Valuation** $\Longrightarrow$ Gives meaning to proposition, i.e. a proposition is said to be meaningful if its validity can be assessed.
- **Meaningless propositions** :
  - "The unicorn is white",
  - "A or and B" $\Rightarrow$ Statement which violates the rules of the logic in use.

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Valuations in Classical physics

- **Propositions** $\Longrightarrow$ Identified with subsets

$$(X \in \Delta) = S_{X \in \Delta} = \{s \in S | X(s) \in \Delta\}$$

of the state space S for which they are true.

- **Classical physics is just black and white** $\Longrightarrow$ In Classical physics propositions have definite truth values, they can either be true or they are false

- **Example** Consider a simple one particle problem, where the

particle is confined to move in the z direction.

$$X \in \Delta_z = \begin{cases} \text{True} & \textit{if} \quad X \in \Delta_z \\ \textit{False} & \textit{otherwise} \end{cases}$$

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Valuations in Quantum Mechanics

- **Propositions** $\Longrightarrow$ propositions are identified with subspaces

$$(A \in \Delta) = \mathcal{H}_{\hat{P}_{A\in\Delta}} = \{\vec{\psi} \in \mathcal{H} | \hat{P}_{A\in\Delta}\vec{\psi} = \vec{\psi}\}$$

of the Hilbert space $\mathcal{H}$ for which they are true, therefore proposition $(A \in \Delta)$ can be identified with the spectral projector

$$\hat{E}[A \in \Delta] = \hat{P}_{A\in\Delta}$$

- **Lattice of propositions** Considers a 2 dimensional Hilbert space $\mathcal{H}_2$ and two quantities A and B representing spin in two different directions

$$\hat{A} = a_1\hat{P}_{a_1} + a_2\hat{P}_{a_2}$$
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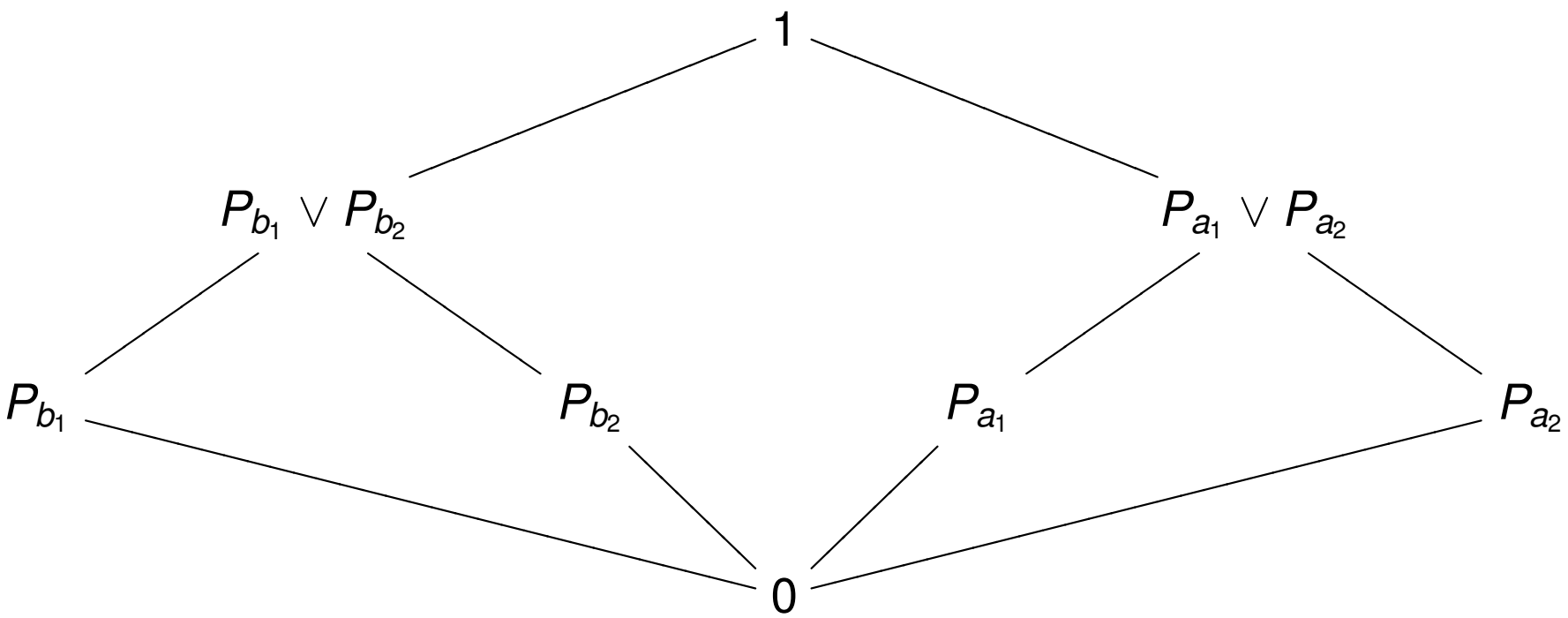

Valuations in Quantum Mechanics

- **Does Classical logic hold at the Quantum level ?$\Longrightarrow$NO**

- **Reasons**

  - **Quantum Mechanics is not black and white** $\Longrightarrow$ In quantum mechanics there exist propositions which are neither true nor false. This propositions usually refer to systems which are in a so called "superposition" state

  - **Quantum Mechanics is contextual**$\Longrightarrow$ The values of certain quantities depends on which other quantities are measured at the same time.

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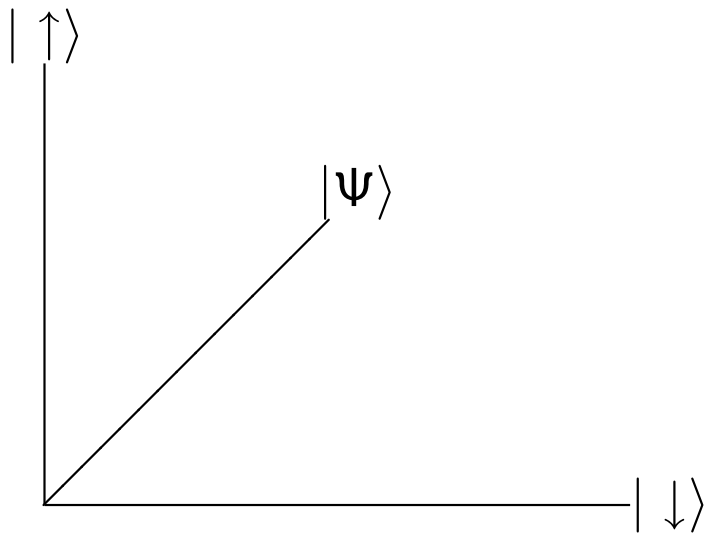

where $|\Psi\rangle = \frac{1}{2}(|\uparrow\rangle - |\downarrow\rangle)$="superposition" of up and down spin

$$|\Psi\rangle \quad has \quad spin \quad up \neq \quad true$$
$$\neq \quad false$$
$$= (?)$$

Valuations in Quantum Mechanics

- Consider four operators A, B, C and D such that [A,C]=[B,D]=0. A and B have the following spectral decomposition:

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- $\hat{P}$ can be expressed in terms of $\hat{A}$ or of $\hat{B}$:

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Why do we need well defined Valuations in Quantum Mechanics?

- **PHILOSOPHICAL MOTIVATIONS**:
  - **As it stands Quantum Mechanics is meaningless** $\Longrightarrow$ The impossibility of assigning truth values to Quantum propositions in a coherent way renders the theory meaningless
  - **Quantum theory needs an interpretation** $\Longrightarrow$ What is the picture of reality Quantum Mechanics suggests? Why is there such a gap between the Logic of classical mechanics and the Logic of Quantum Mechanics?
- **PHYSICAL MOTIVATIONS**:
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Redefinition of valuations in Quantum mechanics

- **Problem**:
  The Black and White view given by classical Logic is not sufficient. Quantum Mechanics need a logic which is

- **Multi-valued**

- **Contextual**

- **Solution**: **Intuitionist Logic defined using Topos Theory** $\Longrightarrow$
  The notion of contextuality and multi-valued arise naturally in the context of Topos theory.

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  Truth values of propositions in Topos theory are identified with the set of generalisations (coarse grainings) of that propositions which are classically true.$\Longrightarrow$ Truth values become more than gust true or false.

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  Contextualisation arises from the fact that the set of generalised propositions (coarse grainings) for a given proposition form an algebra which is specific to that proposition.

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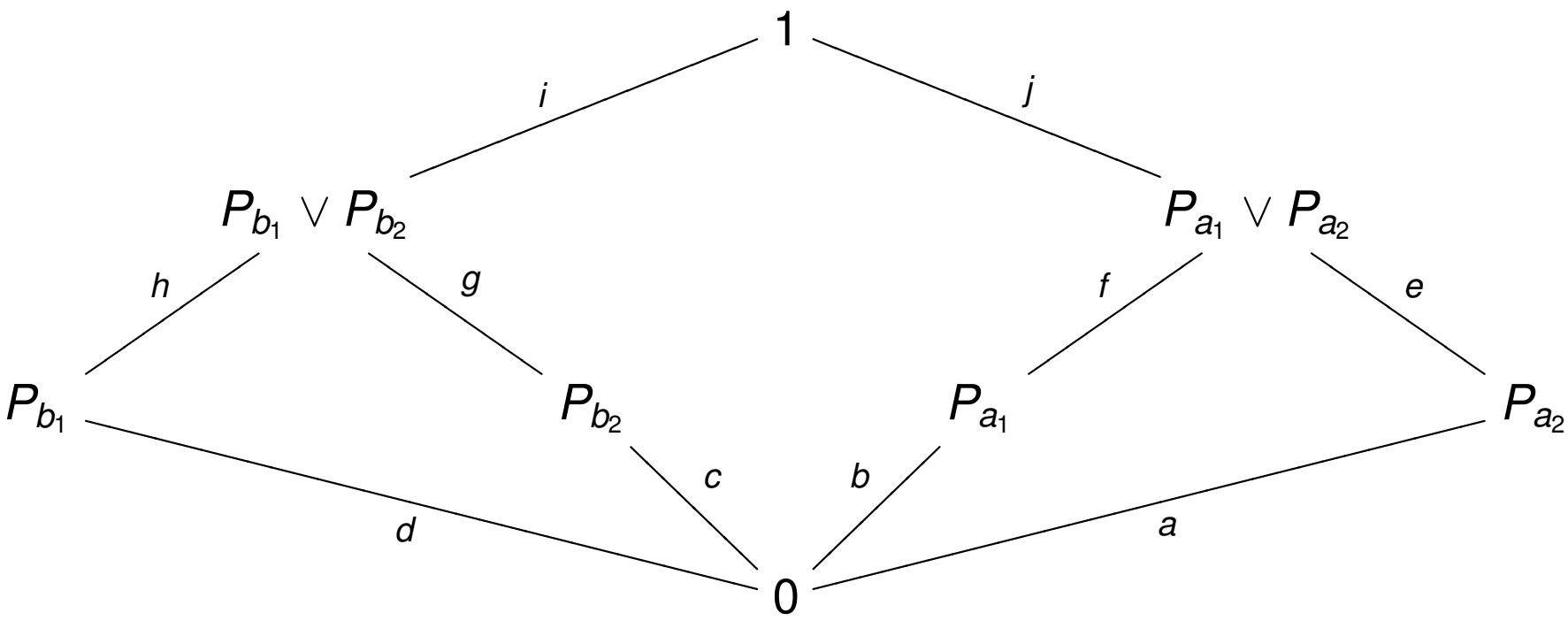

"$|\Psi\rangle (= b_1 + b_2)$ has spin b_1 in the B direction" $= \{h\}$

"$|\Psi\rangle$ or $|\Phi\rangle (= a_1 + a_2)$ have spin b_1 or a_1 in B and A direction" $= \{h, i, f, j\}$

What have we achieved by this Redefinition

- We are now able to give meaning to (?) $\Longrightarrow$ Quantum Theory is not meaningless
- Real Logic is Intuitionist Logic of which classical is a limit, i.e. Limit in which superimposed states do not arise $\Longrightarrow$ No gap between classical Logic and Quantum Logic
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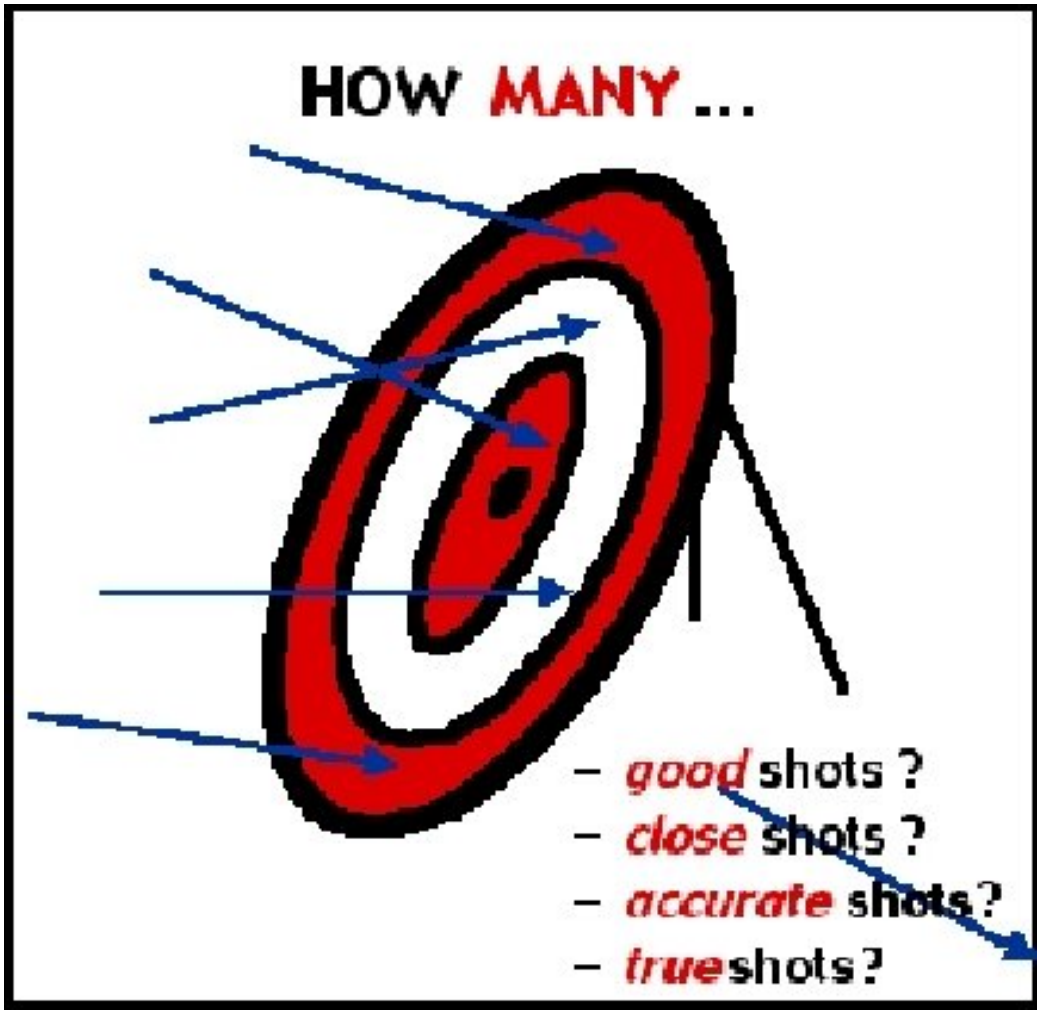
HOW MANY ...
– good shots ?
– close shots ?
– accurate shots?
– true shots?